PROBLEMS AND SOLUTIONS.

B. F. FINKEL, CHAIRMAN OF THE COMMITTEE.

PROBLEMS FOR SOLUTION.

Special Notice.—Please reread the requests as to form of solutions on pp. 258–259 of the October 1913 issue. Unless these directions are observed by contributors, solutions must either be entirely rewritten by the committee or else rejected. Put all drawings on separate sheets.

MANAGING EDITOR.

ALGEBRA.

When this issue was made up no solutions had been received for numbers 396 to 400 inclusive. Please give attention to these.

401. Proposed by R. D. CARMICHAEL, Indiana University.

Prove the validity of Borda's series:

$$\log (x+2) = 2\log (x+1) - 2\log (x-1) + \log (x-2) + 2\left[\frac{2}{x^3-3x} + \frac{1}{3}\left(\frac{2}{x^3-3x}\right)^3 + \frac{1}{5}\left(\frac{2}{x^3-3x}\right)^5 + \cdots\right].$$

402. Proposed by R. D. CARMICHAEL, Indiana University.

Obtain other series similar to that of Borda, given in the preceding problem.

403. Proposed by C. N. SCHMALL, New York City.

A torpedo-boat 40 miles from shore strikes a rock, making a rent in her hull which admits water at the rate of 15 tons in 48 minutes. The ship's pumps can expel 12 tons in an hour. If 60 tons of water is sufficient to sink the boat, find the average rate of steaming so that it may reach the shore just as it is about to sink.

404. Proposed by V. M. SPUNAR, Chicago, Illinois.

Show that $(a+b)(a+b-1)\cdots(a+b-n+1) = a(a-1)(a-2)\cdots(a-n+1) + \binom{n}{1}a(a-1)(a-2)\cdots(a-n+1)b + \binom{n}{2}a(a-1)(a-2)\cdots(a-n+1)b(b-1) + \cdots + b(b-1)(b-2)\cdots(b-n+1)$.

GEOMETRY.

When this issue was made up no solutions had been received for numbers 417, 421 and 425 to 430 inclusive. Please give attention to these.

431. Proposed by F. M. MORGAN, Dartmouth College.

Trisect the angles of the triangle ABC and let the trisectors nearest each side meet in the respective points M, N, P. Prove by trigonometry that the triangle MNP is equilateral.

432. Proposed by ELMER SCHUYLER, Brooklyn, N. Y.

Having given the edges of a tetrahedron, a, b, c, d, e, f, find an expression for the radius of the sphere which is tangent to the six edges.

433. Proposed by W. H. BUSSEY, University of Minnesota.

A transformation of the plane keeping the radius of curvature of all curves invariant is either (1) a real or imaginary motion or reflexion, or (2) not a point transformation.

CALCULUS.

When this issue was made up no solutions had been received for numbers 335, 337, 338, 340, 342, 348, 350, and 352.

353. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

The center of a sphere, radius $R = 5$ inches, is $a = 10$ inches above the surface of a sphere, radius $R = 12\frac{1}{2}$ inches. There is a point of light at $b = 1$ inch horizontally from a point $c = 10$ inches vertically above the surface of the first sphere. What is the area of the shadow which the upper sphere casts on the lower one?

354. Proposed by C. N. SCHMALL, New York City.

Prove $\Gamma(1 + x)\Gamma(1 - x) = \dfrac{\pi x}{\sin \pi x}$.

355. Proposed by C. N. SCHMALL, New York City.

Given the curve of the nth degree,

$$y^n - (a + bx)y^{n-1} + (c + dx + ex^2)y^{n-2} + \cdots = 0,$$

show that if each ordinate is divided by the corresponding subtangent, the sum of all the resulting ratios will be a constant.

MECHANICS.

When this issue was made up no solutions had been received for numbers 271, 272, 275, 277, 278, 279, and 282.

286. Proposed by C. N. SCHMALL, New York City.

A slightly elastic string is just long enough to reach between two hooks on the same horizontal line. A ring of weight w is placed at its middle point. Show that the ring will sink through a distance $h = a\sqrt[3]{3ew/2}$, where e is the elasticity of the string and $2a$ the distance between the hooks.

287. Proposed by WALTER H. DRANE, Lebanon, Tenn.

While sitting in an empaled enclosure, I noticed that the spokes of the wheels of passing automobiles, when viewed through the pickets of the fence, appeared to revolve more slowly than they really did, and in some instances even appeared to be revolving in a direction opposite to that in which they were really turning. Explain this optical illusion.

NUMBER THEORY.

When this issue was made up no solutions had been received for numbers 187, 189, 191, 192, 194, 201, and 202 inclusive.

205. Proposed by E. T. BELL, New York City.

Show that in the usual arithmetical sense the form that follows admits of composition; give the requisite transformations, and indicate how several (if not all) solutions may be found. The variables are the x_i.

$$x_0^2 + nrx_1^2 + mrx_2^2 + mnx_3^2 + mnrx_4^2 + mn^2r^2x_5^2 + nr^2m^2x_6^2 + rm^2n^2x_7^2.$$

206. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that the sum of the sixth powers of two integers cannot be the square of an integer.

207. Proposed by A. J. KEMPNER, University of Illinois.

There are 80 positive integers < 100 containing no figure 9 against 19 containing at least one figure 9. (For integers < 1000 the numbers are 728 and 271 respectively.) One might be led to believe that for every positive integer M the number N_1 of positive integers $< M$ containing no figure 9 is always greater than the number N_2 of positive integers $< M$ containing at least one figure 9.

To prove: $\lim\limits_{M \doteq \infty} N_1/N_2 = 0$. See pages 48–50 of January, 1914, issue.